Erratum: Vortex core dynamics and singularity formations in incompressible Richtmyer-Meshkov instability [Phys. Rev. E 73, 026304 (2006)]

Chihiro Matsuoka and Katsunobu Nishihara

(Received 29 May 2006; published 5 October 2006)

DOI: 10.1103/PhysRevE.74.049902 PACS number(s): 47.32.C−, 47.20.Ma, 02.70.Pt, 47.11.−j, 99.10.Cd

We found that we have used a wrong value for one coefficient in equations in our program for numerical calculations with the regularized parameter $\delta=0$, therefore, some differences have been caused in part of the results, especially for the Atwood number 1.0 in Figs. 8–12 in Sec. IV (RM instability) and Figs. 15–18 in Sec. V B (RT instability). In the original calculations for $\delta=0$, the absolute values of velocities of bubbles and spikes at time $t=0$ are not equal to one, the expected value from the theoretical result.

We corrected those figures in this erratum. Most of the figures are seemingly unchanged, however, we show in this erratum all figures (Figs. 8–12 and Figs. 15–18) calculated with corrected parameters. The figures where the differences clearly appear are Figs. 10(b), 11(b), 12, 16(d), and 17(b) in the original paper, which are related with the calculations for $A=1.0$. The conclusion for these calculations is also unchanged, however, we have rewritten the relevant part to corrected figures in Sec. VI (Discussions and conclusion).

The numerical parameters adopted in the calculations are as follows. The time steps are set to $\triangle t=1.0\times 10^{-4}$ ($\triangle t=2.0\times 10^{-6}$ in the original paper) for Figs. 8–12 and $\triangle t=5.0\times 10^{-4}$ for Figs. 15–18 ($\triangle t=1.0\times 10^{-4}$ in the original paper), respectively. The number of grid points N and the artificial parameter α are set to $N=1024$ and $\alpha=-A$, A, the Atwood number, for all calculations (same as those in the original paper). The section and figure numbers in this erratum correspond to those in the original paper.

IV. SINGULARITY FORMATIONS IN RICHTMYER-MESHKOV INSTABILITY

The following three paragraphs replace the first three paragraphs on p. 10 of the original paper.

Figure 8 shows amplitudes of the Fourier coefficients $|\hat{C}_m(t)|$ versus mode number, where dashed lines in the figure have slope $-5/2$, i.e., the value predicted by Moore for the KH instability. The spectra approach to $-5/2$ lines as time t approaches

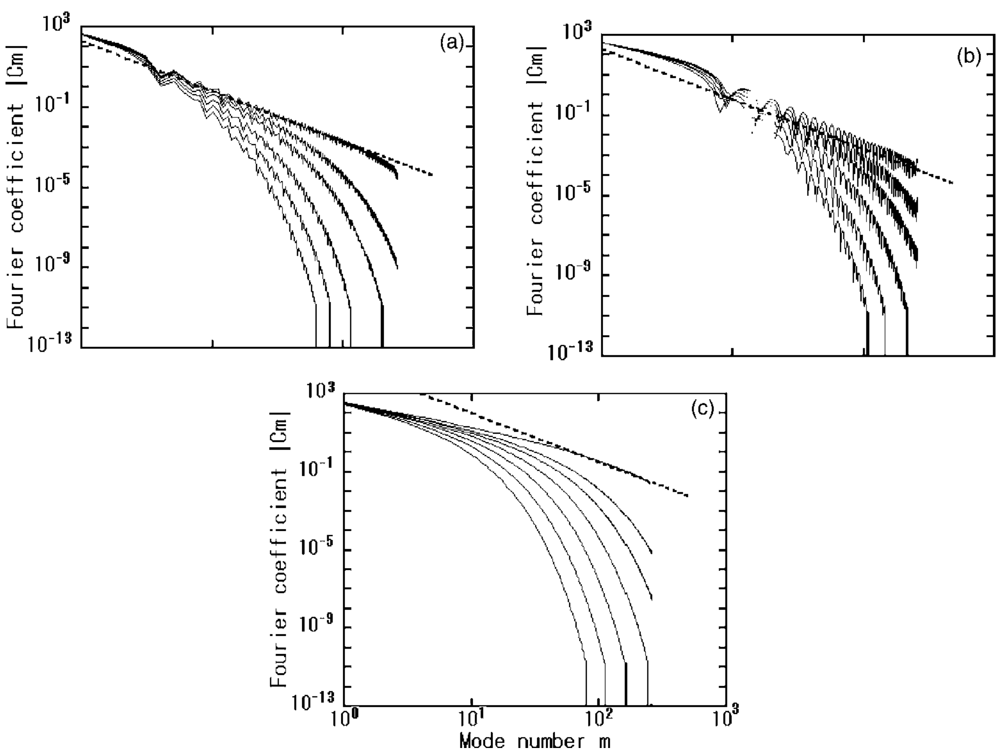

FIG. 8. Log-log plots of the Fourier coefficients for (a) $A=0.155$ at time $t=0.68$, 0.74, 0.80, 0.86, 0.89, 0.92, (b) $A=0.5$ at $t=0.70$, 0.79, 0.85, 0.90, 0.93, 0.96 and (c) $A=1.0$ at $t=0.36$, 0.42, 0.48, 0.54, 0.60, 0.65, 0.80. The slope of the dashed line is $-5/2$, the value obtained by Moore for the KH instability.

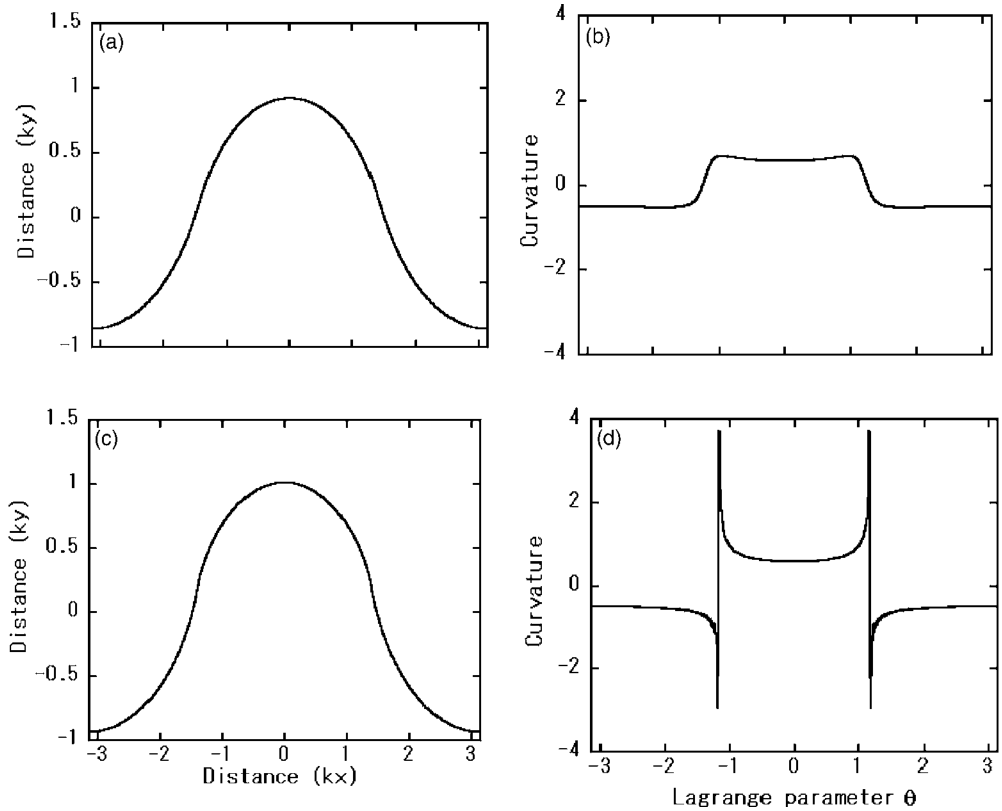

FIG. 9. Interfacial profiles and the curvatures for $A=0.155$ at $t=0.80$ [(a) and (b)] and $t=0.92$ [(c) and (d)].

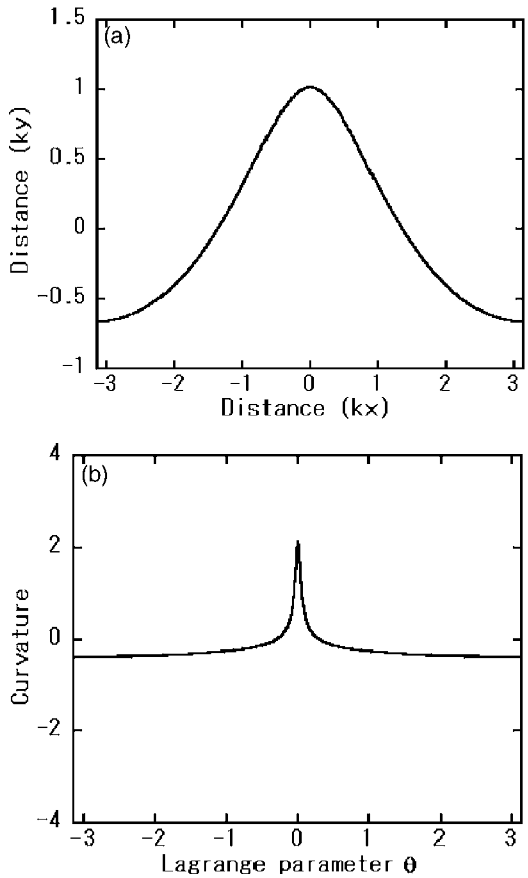

FIG. 10. Interfacial profile and the curvature for $A=1.0$ at $t=0.65$.

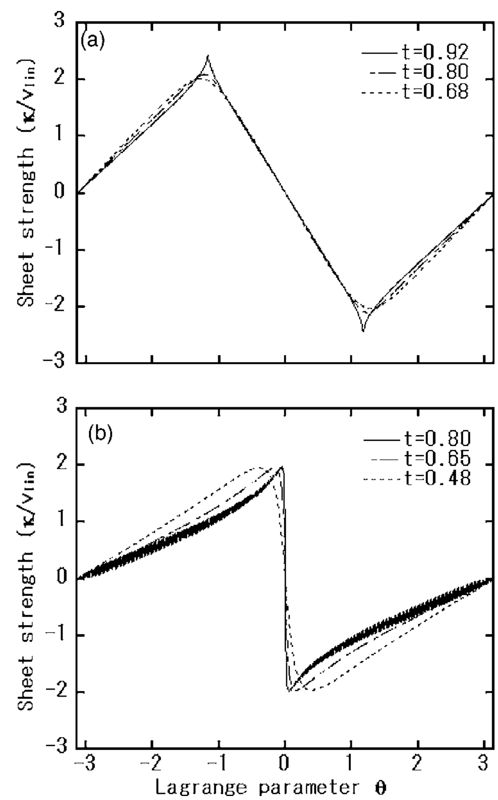

FIG. 11. Sheet strength κ in the RM instability at $\delta=0$ for $A=$ (a) 0.155 and (b) 1.0, where the dashed, dashed-dotted, and solid lines depict $t=0.68$, $t=0.80$, and $t=0.92$ in (a), while they depict $t=0.48$, $t=0.65$, and $t=0.80$ in (b), respectively.

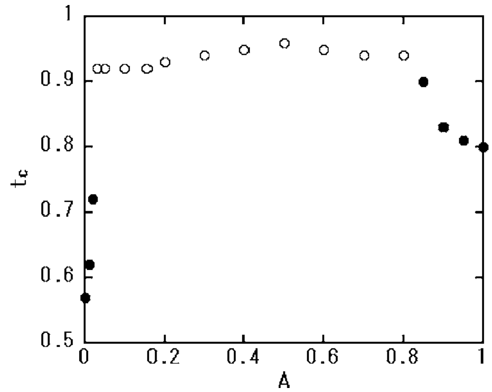

FIG. 12. Critical time t_c in the RM instability for various Atwood numbers, where black circles ($A<0.03$ and $A>0.85$) denote the parts in which some oscillations are involved as $t\rightarrow t_c$.

the critical time t_c, although the interval which the Fourier spectrum at $t=t_c$ fits the $-5/2$ line becomes shorter as the Atwood number becomes higher, where $t_c=0.92$ ($t_c=4.49\times10^{-3}$ in the original paper), $t_c=0.96$ ($t_c=4.20\times10^{-3}$ in the original paper) and $t_c=0.80$ ($t_c=3.62\times10^{-3}$ in the original paper) for $A=0.155$, $A=0.5$ and $A=1.0$, respectively.

Interfacial profiles and curvatures for $A=0.155$ at $t=0.80$ ($t_c=4.40\times10^{-3}$ in the original paper) and $t=0.92$ ($t_c=4.49\times10^{-3}$ in the original paper) are shown in Fig. 9. The interfacial profile at (c) $t=0.92$ is smooth, however, two discontinuities in the neighborhood of $\theta=\pm1.4$ appear in the curvature profile. After a few time steps of this critical time, the curvature of the vortex sheet diverges and the computations break down. These two discontinuities gradually approach and form one peak at $\theta=0$ for higher Atwood numbers [see Fig. 10(b)].

The explanation of the following figure is rewritten considerably.

Figure 10 shows the interfacial profile and curvature for $A=1.0$ at $t=0.65$. A sharp peak is observed at the spike $\theta=0$ in the curvature profile 10(b). This peak value at $\theta=0$ is almost unchanged up to the critical time $t_c=0.80$, i.e., time at which the Fourier coefficient fits the $-5/2$ line [see Fig. 8(c)], however, a violent numerical oscillation appears on both sides of the peak as $t\rightarrow t_c$ and the amplitude of the oscillation exceeds the peak value at $t=0.80$. This numerical oscillation begins to appear in the curvature profile when the Atwood number $A\geq0.95$ and $t\rightarrow t_c$. This suggests that it is difficult to calculate higher order derivatives in the neighborhood of $t=t_c$ numerically when the Atwood number is close to 1.0.

In Fig. 11, we show the sheet strength κ for several times, where the solid lines (a) and (b) depict the critical sheet strength $\kappa(\theta,t_c)$ for the Atwood numbers. We see that the sheet strength for $A=0.155$ forms two cusps at critical time $t_c=0.92$. The existence of cusps in κ for $A=0.155$ suggests that the sheet strength for this Atwood number has the form of $\kappa\sim|\theta|^{\beta'}$ in the neighborhood of cusp points for some $\beta'<1$ as $t\rightarrow t_c$, as analogous to the KH instability case. Such cusps are not so clear for $A=1.0$, which suggests that the singularity formations may not occur for this Atwood number, even though the Fourier spectrum fits the $-5/2$ line. Generally, as the Atwood number increases, the two cusps approach to each other and they form a sharp discontinuity at $\theta=0$, where the curvature also has a sharp peak.

In the following paragraph, which replaces the first paragraph on p. 11 in the original paper, the explanation especially for high Atwood numbers ($0.85\leq A\leq1.0$) is rewritten.

Dependence of the critical time t_c on various Atwood numbers is depicted in Fig. 12. The critical time t_c takes (almost constant value) 0.92–0.96 for $0.03\leq A\leq0.83$ (white circles), however, it rapidly decreases with the decrease of the Atwood number for $A<0.03$ (black circles). The decrease is also found for $0.85\leq A\leq1.0$ (black circles), although the rate is not as rapid as the one for $A<0.03$. For these two intervals $A<0.03$ and $A>0.85$, some oscillations appear in the calculations as $t\rightarrow t_c$. It is not clear at present that these oscillations are caused by the numerical instability or the solution itself has some oscillation part, therefore, we mention here that there is some ambiguity in the determination of the critical time for those ($A<0.03$ and $A>0.85$) Atwood numbers.

V. CORE STRENGTH AND SINGULARITY FORMATIONS IN RAYLEIGH-TAYLOR INSTABILITY

B. Singularity formations in Rayleigh-Taylor instability

The following four paragraphs replace the third through sixth paragraphs on p. 12 of the original paper.

In Fig. 15, we show amplitudes of the Fourier coefficients $|\hat{C}_m(t)|$ versus mode number m, where dashed lines in (a) and (b) in the figure have slope $-5/2$. The spectra approach to the $-5/2$ lines as time t approaches critical time $t_c=6.47$ ($t_c=0.4640$ in the original paper) for $A=0.155$ and $t_c=2.55$ ($t_c=0.1684$ in the original paper) for $A=1.0$. The Fourier spectra in the RT instability also fit to Moore's $-5/2$ power law for various Atwood numbers, although the interval which the Fourier spectrum at $t=t_c$ fits the $-5/2$ line becomes shorter as the Atwood number becomes higher, as found in Fig. 8.

[In the following paragraph, the explanation of Fig. 16(d) is rewritten considerably.]

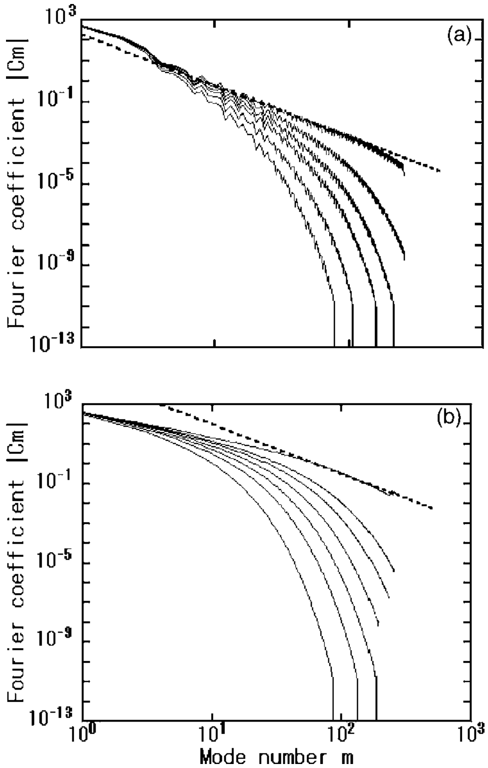

FIG. 15. Log-log plots of Fourier coefficients for A= (a) 0.155 and (b) 1.0, where plotted time is t=6.00, 6.15, 6.28, 6.34, 6.41, and 6.47 in (a) and 1.83, 1.98, 2.07, 2.15, 2.23, 2.30, and 2.55 in (b), respectively. The dashed line in the figure has slope $-5/2$.

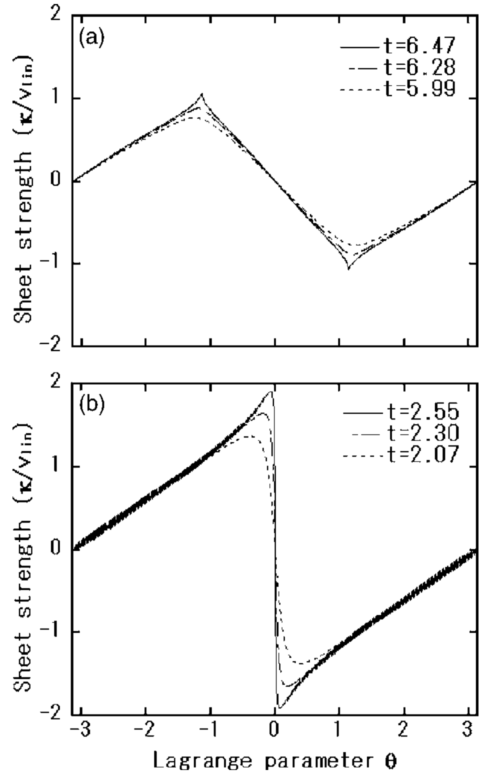

FIG. 17. Sheet strength κ in the RT instability at δ=0 for A= (a) 0.155 and (b) 1.0, where the solid, dashed-dotted, and dashed lines depict t=6.00, 6.28, and 6.47 in (a), while t=2.07, 2.30, and 2.55 in (b), respectively.

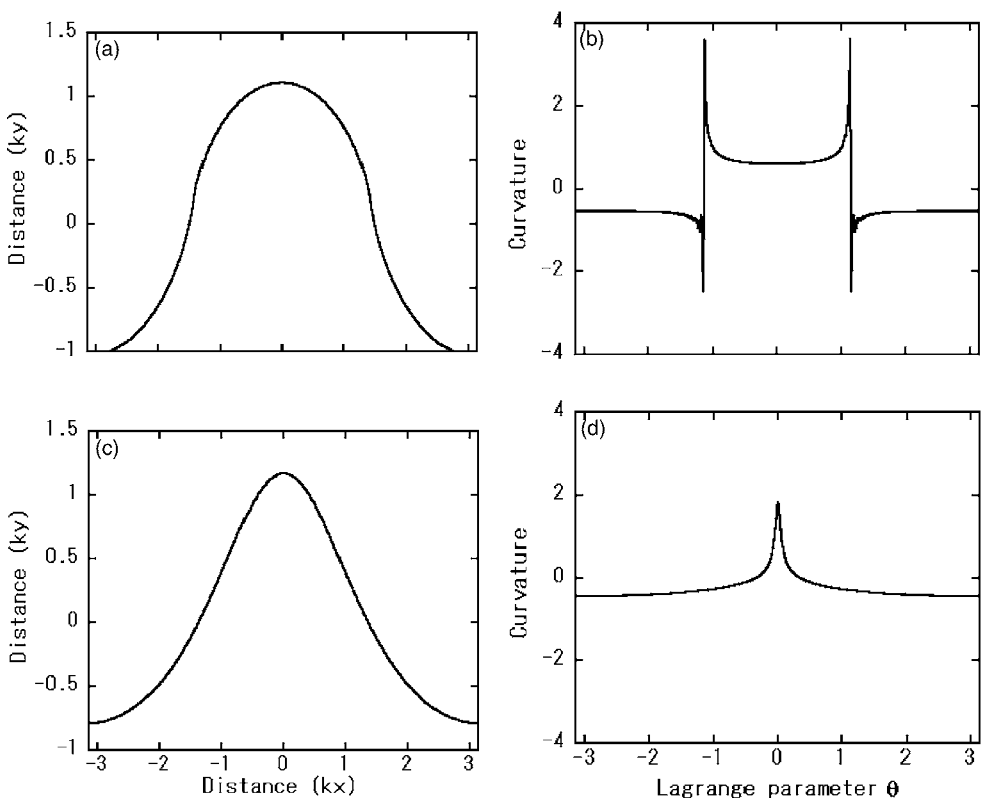

FIG. 16. Interfacial profiles and curvatures at t=6.47 for A=0.155 [(a) and (b)] and at t=2.30 for A=1.0 [(c) and (d)], where (b) and (d) are curvature profiles of the interfacial profiles (a) and (c), respectively.

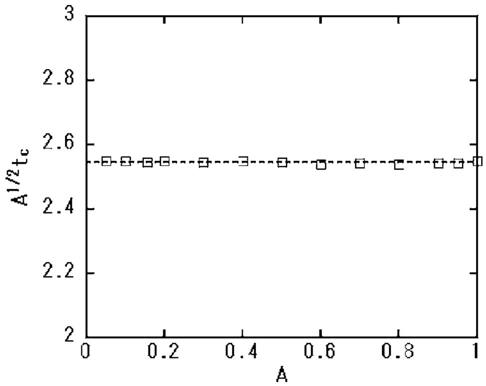

FIG. 18. Critical time $\sqrt{A}t_c$ in the RT instability for various Atwood numbers, where squares denote our numerical calculations, while the dashed line gives the theoretical prediction by Baker *et al.*

Interfacial profiles and curvatures at $t_c=6.47$ for $A=0.155$ and $t=2.30$ for $A=1.0$ are shown in Fig. 16. Time $t=2.30$ is not the critical time for $A=1.0$ [see Fig. 15(b)], however, a violent numerical oscillation as found for the RM instability also appears in the curvature profile for $t>2.30$. From a viewpoint of accuracy of calculations, we have chosen the curvature profile at $t=2.30$.

Figure 17 shows the sheet strength κ up to critical time t_c, where solid lines (a) and (b) depict the critical sheet strength $\kappa(\theta,t_c)$ for the Atwood number. Unlike the RM instability case (see Fig. 11), the amplitude of the cuspidal form for the RT instability is larger for higher Atwood numbers. This may relate to the fact that the strength of a vortex core for $\delta\neq 0$ is larger for higher Atwood numbers. Analogous to the RM instability case [see Fig. 11(b)], cuspidal form for $A=1.0$ is not as clear as that for $A=0.155$.

[There is no change in Fig. 18 seemingly, however, we have rewritten the relevant paragraph with the corrected figure.]

Dependence of the critical time t_c on the Atwood numbers is depicted in Fig. 18, where the dashed line and squares denote the theoretical prediction by Baker *et al.* [1] and our numerical computations, respectively. When $A\neq 0$, the critical time t_c in the RT instability is estimated by Baker *et al.* [1] as

$$t_c = \frac{\tilde{C}}{|Ag|^{\frac{1}{2}}}, \tag{14}$$

where the constant $\tilde{C}$ generally depends upon the initial conditions in the computation. Our numerical computations support this theoretical result, although it is unclear whether the curvature singularity occurs or not for $A>0.95$ due to the numerical oscillations stated above. Both Baker *et al.* [1] and Tanveer [2] predict that some singularity exists in the complex, i.e., unphysical plane, however, it will never reach the real axis, i.e., physical plane. It is not easy to verify this prediction for $A=1$ by numerical computations as pointed out in Ref. [1].

VI. DISCUSSIONS AND CONCLUSION

We rewrite part of discussions and conclusion for the singularity formations; this paragraph replaces the first paragraph on p. 14 of the original paper.

The cuspidal formations in the sheet strength κ when $t\to t_c$ are vague for $A=1.0$ for both RM and RT instabilities and the cusp-like structures appear at spikes, not at the points which the vortex cores are expected to be formed when $\delta\neq 0$. The maximum values (peaks) in curvature profiles for $A=1.0$ are also found at spikes for both RM and RT instabilities and there exist violent oscillations in the neighborhood of the points which the vortex cores are expected to be formed. In addition to that, the computations for $A>0.95$ for both RM and RT instabilities do not break down in short time as found in the case for lower Atwood numbers when time t exceeds the critical time t_c, where we have defined it as the time at which the Fourier spectrum fits the Moore's -5/2 power law, the sheet strength κ has cuspidal form and the curvature profile has sharp discontinuities. These may suggest that the singularity formations for $A=1.0$ does not occur in finite time for both RM and RT instabilities.

[1] G. Baker, R. Caflisch, and M. Siegel, J. Fluid Mech. **252**, 51 (1993).
[2] S. Tanveer, Proc. R. Soc. London, Ser. A **441**, 501 (1993).